Quality-factor considerations for single layer solenoid reactors

M. Nagel[1*], M. Hochlehnert[1] and T. Leibfried[1]

[1]University of Karlsruhe, Institute of Electrical Energy Systems and High-Voltage Technology

Engesserstr. 11, 76131 Karlsruhe, Germany

*Email: nagel@ieh.uni-karlsruhe.de

Abstract: As it is commonly known, rating and building a reactor is not the easiest thing to do. Numerical solutions of complex magnetic field distributions need to be calculated to verify the design. This is getting even more challenging, when the quality-factor (Q-factor) of a resonator is of major importance. This problem is for example appearing, when building resonance circuits with low losses for high voltage and high frequency (HF) experiments. Coreless, one layer solenoid, or likewise, reactors are needed there to meet both frequency and voltage requirements. Coreless because of the hysteresis losses of magnetic materials at high frequency and only one layer, due to the maximum applicable field strength per winding, which is dropping significantly at high frequency voltage stress.

Several considerations on the design of the reactor were done and compared with real implementations of those solenoid reactors. Initial boundary conditions are a fixed reactance needed and the length of the wire that needs to be as short as possible. Still variable are the diameter and length of the coil, the conductor thickness and type of wire, meaning normal enamelled- or special HF-litz wire. While the latter influence is easy to determine, the higher the frequency, the more it is advised to cope with the skin-effect and to use HF-litz, all other factors are bound to the magnetic induction of the construction. To avoid complex field simulations, a close observation of the magnetic field at discrete positions is derived into a normalized Q-factor-curve for the evaluation of different configurations. Measurements of several model reactors were used, either to assess the found influences and to verify the gathered curve. A close prediction of the Q-factor to be measured at a newly build solenoid at a fixed frequency could thereby be performed. The result of these considerations is an optimum sizing of a solenoid reactor and an estimate on the possible loss of quality for a deviant design.

1 INTRODUCTION

Driven by the continuing rise of energy consumption in the industrial centres of modern cities, the need for more long range transportation of energy is getting more urgent. Along side with the prospect of decentralised generation, e.g. by solar, wind and wave generation the need for transportation capability is gaining more and more priority, not to mention the grid connection of those generation plants. As already discussed, for example by cigré [1], those changes in existing networks are affecting the performance of existing equipment. High frequency noise is subjected to components, previously only fed with clean sine waves of 50/60Hz.

This noise, mainly superimposed by power electronic equipment like used in Flexible AC transmission systems (FACTS) build with IGBT valves, is thereby a new stress to the equipment and poorly investigated yet. While in the above mentioned case, the large slew rates of the switching pulses is generating the noise, more longer known is high frequent noise to arise by transformer switching over longer lines.

In common for all the cases of high frequency noise at standard equipment is the present lack of knowledge about the performance of the insulation system. In particular oil or paper insulations of power transformers, being the main cause for investigations in that field [2].

The first step in any consideration about testing materials is the building of a suitable source. In this case this means Voltage amplitudes up to 100kV at frequencies between 10kHz and 250kHz. The higher the frequency can be driven, the better and the more results are possible to obtain.

2 REACTOR IN TEST SETUP

For the tests described above, concerning high frequency and high voltage testing of dielectric materials, different basic setups of the power supply can be considered. The only one suitable for the tasks of producing a constant sine wave with variable amplitude is a series resonating circuit. This namely consisting of a capacitor and a reactor connected in series with an excitation source as it can be seen in Fig. 1.
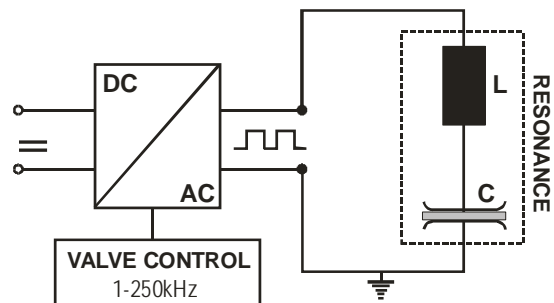

Fig. 1: Resonance circuit in the basic test setup

Hereby the excitation voltage is provoking the oscillating circuit to work at its natural frequency. By the driven current in the circuit, a voltage drop-of over the

reactance is producing the desired amplitude for the measurements. The specimen under investigation is put between the electrodes of the capacitor and stressed with that same voltage, which is also recorded.

There are three major boundaries in this whole setup. First, the resonance frequency of the series circuit needs to be the one being under test. A task not too easy to fulfil, concerning the dielectric constant of the used material and the impact on the electric field in the capacitor. A tuning of the circuit is therefore absolutely necessary. This can either be done by varying the reactance, a suggestion which is practically not easily done for low frequency increments, or by adding or subtracting more capacitance.

Secondly and surely the most annoying in the design process is the constant use of materials, that are not investigated for the kind of stress being subjected to them. For example is a reactor in use, for which you need to design the number of turns, layers, insulation and thickness. Some of the limitations are easy to make. A roughly calculated reactance, corresponding to the minimum capacitance, will give basic clues. The test voltage rising up to 100kV will result in the same voltage amplitude between the ports of the reactance. If not using a single layer, this voltage is present in a winding next to another. This can be mastered by shaping the insulation type and thickness. At least as long as standard AC or DC stress is used. Calculating thicknesses, amounts and behaviours with unknown values is simply waste of paper and time.

Thirdly, and initially causing this presentation, is the reachable amplitude in the circuit, directly bound to the quality factor Q of the elements the higher the frequency gets.

$$Q_{RC} = \frac{1}{R} \sqrt{\frac{L}{C}} \qquad (1)$$

The actual resistance R, compared to the inductive (L) and capacitive (C) part is dampening and limiting the reachable resonance amplitude. Given a fixed excitation voltage. The above stated problems are concerning both elements equally. Yet the capacitor consists in a large amount of the specimen under investigation which can not be changed. Tuning is done by adding ceramic capacitors, connected in series for not exceeding their estimated breakdown strength. No shaping of quality factor can be done in either of the capacitors. Only by the design and shape of the built reactance, any positive change in the quality factor can be achieved.

Cores inside the reactor are totally neglected here, since only ferrite materials could handle such high frequencies, but being priceless for such big diameters and volumes of the needed one. Normally used iron plated core materials would simply be in complete saturation and physically impossible to use.

Concerning all those obstacles, only single layer reactors are investigated closely. Yet some of the found resemblances and factors for compassion of different quality factors seem to be valid for different designs and even between them as well.

3 INDUCTANCE AND QUALITY

When looking at the investigated reactance we need to focus on the actual inductivity of it and its calculation. According with that, it is broadly known, that a simple formula for each design is not available, nor anything usable available. All considerations on that matter, suitable for simple working formulas are given and derived by [3]. Main focus for any of those calculations is always a given setup of windings, finally leading to a inductivity L. Anything of that is done by using different boundary considerations, only usable in that particular case. This is neglecting the fact of any quality of the designed dimensions.

Considerations on any criteria of the design is done for transformer and converter cases, or for radio frequency considerations in how to obtain the largest inductance value possible with a given wire.

Solving ways like those are not feasible in our case. Given by the resonance circuit, a fixed value is needed crucially. Starting from there we need to build the best reactance possible. Only boundary there is the building of it.

So what is influencing the quality of a reactor, given by (2) with the used frequency f of investigation.

$$Q_{\text{Reactor}} = \frac{2 \cdot \pi \cdot f \cdot L}{R_L} \qquad (2)$$

Since, as stated above, the frequency and Inductivity are not changeable, only by reduction of the real Resistance of the reactor, a rise in quality-factor is possible. So it is absolutely needed to think about what is in total causing R_L.

First to mention there is the wire itself. The used material, copper in most cases, has a real resistance. While we are not changing the material or distributor this value is unchangeable either. In any case this effect is present. Which means equally for either DC or AC phenomenon.

At high currents the skin effect will also influence the resistance of the wire. Any conducted current will see a displacement towards the surface. This is leading to a higher current density in the hull and less to none in the middle. Even more displacement is occurring when the frequency of the used current rises. Ultimately, the higher the surface area of any used conductor is, the lower will be its resistance and vice versa.

Not only are we solely looking at the wire itself, but to the linking of the windings to one another. Thereby the emitted magnetic field of all wires to each other is becoming of interest. Namely the proximity effect is causing a change of conduction area in a wire and therefore increasing the resistance.

All of those effects together are causing the quality factor of any given coil at its measured value. Yet the

boundary conditions of our problem here are supporting a little reduction of the many causes and unknowns.

Beginning with the wire itself. For high frequencies a special HF-litz wire can be used, having many insulated strings that are ultimately increasing the surface area. The use of the wire itself is increasing the quality. Yet the amount, length and type of winding is not playing a significant role in any mathematical description of this effect. All of it can thereby be take as a constant factor, constantly adding to the whole. So no matter what we will do, any coil built with HF-litz wire will have a higher quality factor than the one without.

Nearly the same applies to the mere diameter. With the use of a larger diameter of wire, the quality of the resulting coil will rise ultimately. Therefore a simple fact as well and no optimization possible there and giving in our case the simple rule to use the biggest diameter possible and reasonable.

Despite all those effects, which were just excluded, several need to be viewed more closely and are leading to the theoretical quality considerations, allowing the comparison of coils.

4 THEORETICAL QUALITY-FACTOR

The remaining major factor, affecting the current distribution inside the given wire is solely the magnetic field, existing at any given point of it.

Independent of any changes we can define the current as 1 in the wire. Therefore the same current needs to run through every part of the coil windings itself. Each of those defined currents will produce a magnetic field, due to the Biot-Savart law [4]. Starting from the middle of the wire in one linear direction r, there is a linear rise of magnetic field strength H until the radius of the wire is reached. Further in this direction there is a reduction, proportional to $1/r$. The inner field shall be of no interest here, since it is as well equally present in every part of the wire and therefore treated as all the ones excluded before.

Finally the resulting field at any position of the windings x, produced by all the other winding parts is important. Since the originating current is always the same, only the geometric distance is important and therefore leading to the resulting magnetic field. Written in complex values in a rotationally symmetrical coordinate system, every magnetic field will be:

$$\underline{H}_{norm}(x) = \sum_{n \neq x} \pm \frac{1}{\underline{Pos(Winding\_n)} - \underline{Pos(x)}} \quad (3)$$

For the calculation important is the direction of the current, originating that magnetic field. Similar directions of current therefore give the same sign in front of that part of H_{norm}. Thereby a value of the magnetic field is given for every part of the coil, discarding all material properties of the gap in between.

Now, at any of the windings a resulting magnetic field in any direction will occur. This will lead to a dis-

placement of current in that particular wire accordingly. Considering the conductor to be homogeneous, the actual direction of change is not important and can be discarded. The absolute values of the resulting complex magnetic vectors is therefore sufficient. A field distribution at the wire positions in our section plane is thereby the result.

The theoretically calculated distributions now need to be linked in total to the quality factor of real coils. In order to do that and at the same time adding some realism to the done calculations, model coils are built and measured for comparison. A factor needs to be found, that is linking the field distributions, origin for displacement currents and with it quality factor, to the actual quality factor for the total coil.

Three models are built to pay tribute to different building factors that can perhaps affect quality. Since different inductivities affect the measured quality factor, no change is allowed there and kept as close as possible at $L=177\mu H$. Variations in wire diameter are not tolerable as well and therefore the same wire is used for all model coils. "$C1$" as a cylindrical one layer coil with n=66 windings, a diameter d_1 of 7.5cm and a resulting overall length l_1 of 10.5cm. "$C2$" is built as a double cone like coil, connected at the small diameter end. This should resemble a type of layer winding that can not be used in the real experiments due to voltage strength between the ports of the coil and insulation. Therefore 45° inclination for the cones is used, resulting in 64 windings with d_2 of 4.2cm for the low diameter rising to 11.3cm at the opposite side resulting in an overall length l_2 of 7.1cm. This should increase the linking of the wires to one another, changing the relation of L to R in the design. Last but not least "$C3$" is built using the shortest wire possible for any cylindrical coil. The idea will be, that a shorter wire sees less influence of magnetic fields and will therefore have a better quality. Calculating the needed diameters for a one layer solenoid coil is done backwards from a formula for L, suggested by [5]:

$$L[H] = \pi^2 \cdot \frac{n^2 \cdot d}{0.44 + l/d} \cdot 10^{-9} \quad (4)$$

For the fixed inductance used in this investigation and with a constant diameter of wire, giving the total length when multiplied by the number of windings, (4) can be transformed, calculating the needed diameter. Circumference and turns are giving then the length of wire needing to be minimized.

All of that is resulting in the actual "$C3$" with n=32 windings, a diameter d_3 of 14.1cm and an overall length l_3 of 5.1cm.

When measuring all three coils with an LCR-meter, plotting the quality factor Q, it quickly shows a major difference for the three coils. While 1 and 3 are pretty close together, having $Q_1=131$ and $Q_3=124$, the cone shaped one is only producing $Q_2=70$.

Those differences in measurement results now need to be resembled by the calculated magnetic fields, given

theoretically for the built coils. The field distributions are shown in Fig.2 as the normalized magnetic field by (3) over the positions of the wires.
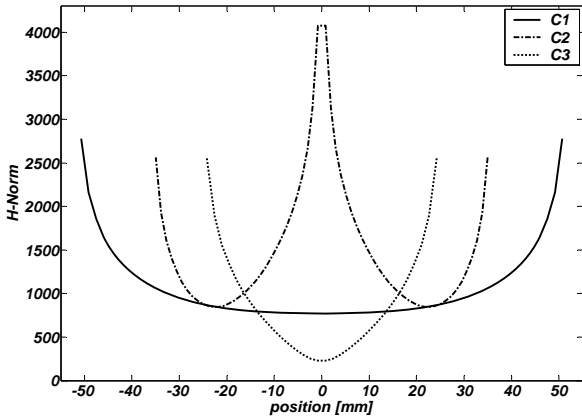

Fig. 2: Magnetic field distribution at the wire positions of the different coils for comparison

Only the combinations of those singular magnetic field strengths will give a resemblance of the complete quality of the coil. But which ones are significant can not that easy be said. The sum of all fields will give no significant value in that matter, as well as the maximum or minimum. Finally the difference (5) of the last two values is showing similar behaviour as the quality-factor.

$$H_{diff} = H_{max} - H_{min} \qquad (5)$$

Taken inversely, the H_{diff} values resemble the measured quality values.

These preliminary investigations show a poor quality for the cone shaped coils. While the newly found criterion H_{diff} suggests a comparison between different building types that should function with any geometry, this was not investigated more closely, since the best quality coil was to be built. Therefore only cylindrical coils were further studied. Benefit of those investigations is an relatively constant calculation of the diameter, based on (4) as shown in Fig. 3.
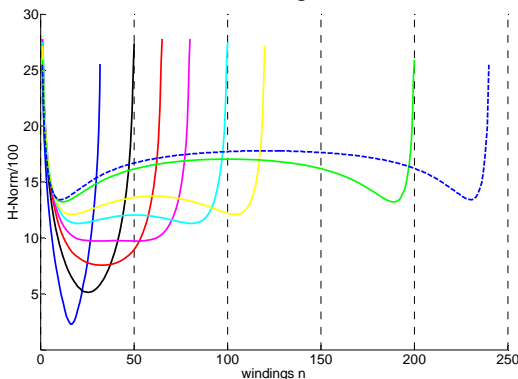

Fig. 3: Magnetic field distribution for different solenoids

An easy calculation of the difference between H_{max} and H_{min} shows a maximum for 32 Windings. Increas-

ing or reducing the number of turns should therefore enlarge the quality-factor.

To verify this assumption a third coil "C4" with n=200 and a diameter of 42mm in cylindrical form is built and measured.

Against all previous predictions the quality reduces to $Q_4=108$, showing that different influences need to be counted in for the quality-factor.

Looking at the various curves in Fig. 3 the major change between few windings and more ones can be seen. There seems not to be a constant magnetic field distribution across all of the windings, but with increasing number of turns a superelevation in the middle of the solenoid is taking place. Ultimately a second factor needs to be found, describing exactly that. Similar to H_{diff}, the difference is taken between the value at the middle and the absolute minimum, leading to (6).

$$H_{superelevation} = H_{middle} - H_{min} \qquad (6)$$

Both thereby found influences need to be weighed, correctly predicting the quality-factor for different solenoids. Additionally it should resemble the behaviour of the quality of a coil, producing a higher value, the higher the quality gets, leading to a theoretical quality factor "TQ" according to (7). For receiving convenient values, the two factors are therefore as well normalized by their maximum.

$$TQ = 1 - \left[\overline{\frac{H_{diff}}{2.5}} + 2.5 \cdot \overline{H_{superelevation}} \right] \qquad (7)$$

The so found formula should now be capable of predicting the measured quality of a solenoid, starting with the known measurement of just one. Since the used factors are found by best fitting the curve to the measured values, a certain mistake is involved there, as well as in differences building the model coils.
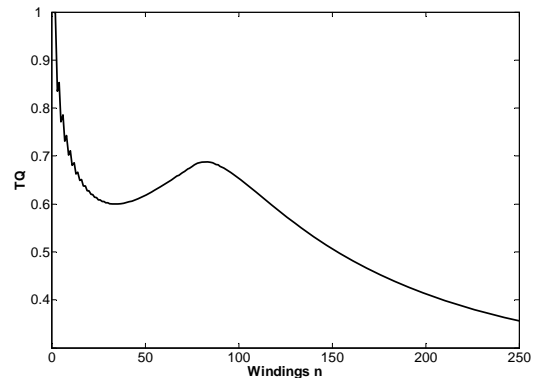

Fig. 4: Theoretical quality factor by (7)

Plotted into a graph, shown in Fig. 4, there can be found a maximum of the *QF*. Since solenoids with only a few turns will have a very large diameter, at least in our case, the 30% increase of the quality of those inductors is discarded in these observations.

Calculations predict, that when building a solenoid reactor "C5" with n=84 turns it should give a measured

quality factor of 138-142. Data from "*C4*" is discarded, since the larger the number of windings is apart, the bigger any mistake will get. Resulting in a predicted quality of 180.

The measurement of the really built coil, having a diameter of 64mm, shows $Q_5=143$. But it is assumed, that without any failures, absolutely same reactances and exactly round solenoids with no air gaps in between the layers, the mistake should be non existent.

More important is the possibility for design guidelines. Since equally all measured and predicted quality-factors are higher than the existing ones, qualitatively the designs can be compared, identifying the better one of both coils.

5 FREQUENCY DEPENDENT TQ

Descriptions about the theoretical quality-factor were only done yet for one frequency of 200kHz, where the measurements with the LCR-meter were performed. Accounting for those differences and finally as well discarding low numbers of turns while normalizing on the possible maximum, the different curves of Fig. 5 originate for frequencies between 100kHz and 300kHz.
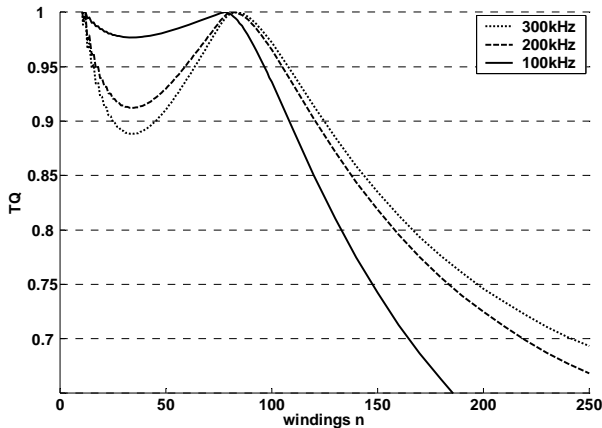

Fig. 5: TQ over windings for different frequencies

As it can be seen, the influence of H_{diff} has a decreasing influence on the actual quality at lower frequencies, while the influence of $H_{superelevation}$ is increasing. For higher frequencies the influence can be seen vice versa.

Again, trying to fit the different curves by a mathematical expression is now generating frequency dependent weighing factors. Thereby the total theoretical quality-factor becomes:

$$TQ = 1 - \left[k \cdot H_{diff} - 2.5 \cdot H_{superelevation} \right]$$

$$k = \frac{21 \cdot \left(\arctan\left(\sqrt{3} \cdot \left(f/10^5 - 1.5 \right) \right) + \arctan\left(1.5 \cdot \sqrt{3} \right) \right)}{32 \cdot \pi} \qquad (8)$$

With this formula, a plot for any frequency is possible. Since it is not based on any theoretical observations, but by curve fitting of real measured values, the more the watched frequency differs form the initial fitting, the higher the error will get.

6 CONCLUSION AND OUTLOOK

Since it can be sometimes a great benefit to get different design specifications for a maximum quality-factor of a solenoid single layer reactor, different models were actually built to evaluate a theoretical basis for that. Therefore the influence on the single wire parts was estimated and linked to the magnetic field distributions present. This was resulting in a theoretical quality-factor.

The same way this was now calculated for the model situation, it can be ported to the coil, actually needed in the initially described experiment This shows exactly how to design the optimum shapes there or to evaluate which attributes should be preferred.

All of the considerations done here for cylindrical coils and their comparison to one another, seem to be as well comparable to other types of geometries. When getting rid of all normalisations and than comparing the results of (8) for cylindrical and the cone shaped *C2,* at least a simple recommendation on which one of the both is better seems to be possible. Taking critical ones like *C2* and *C4*, the values will be $TQ_2=9483$ and $TQ_4=1440$ telling the quality-factor of C4 should be better than the one of *C2*. Measured this is matched with $Q_2=70$ and $Q_4=108$ having an advantage for the cylindrical coil.

Since all of those comparisons were not checked with other designs and model coils it is only a likely assumption. Therefore this can only be a conclusion for now and needs further investigation in that field for further building of reactors with maximum quality-factor.

7 REFERENCES

[1] J. Aubin, S.A.C. Hall, "Colloquium on transformer reliabilityy and electrical transients", Moscow 2005, Electra No.225, April 2006, pp.18-24

[2] M. Nagel, T. Leibfried, "Investigation on the high frequency, high-voltage insulation properties of mineral transformer oil", CEIDP 2006, Kansas City, pp.226-228

[3] F. W. Grover, "Inductance calculations", Instrument society of America, New York, 1973

[4] A. J. Schwab, "Begriffswelt der Feldtheorie" in german, Springer, Berlin Heidelberg, 2002

[5] R. Küchler, "Induktivität und Stromkraft einer Zylinderspule mit Stabkern" in germen, Yearbook of AEG Research, Vol. 6, Springer, 1939