Coupled Electromagnetic and Thermo-Hydraulic Analysis of Current Distribution in Superconducting Cable-in-Conduit Conductors

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Abstract — The performance of high-current superconducting multi-strand cables often lags behind the predictions extrapolated from the performance of a single strand. Non-uniform distribution of the transport current among the strands is likely discussed as a possible cause. In order to investigate current distribution phenomena, a comprehensive model has been developed for the coupled analysis of electromagnetic and thermo-hydraulic processes in forced-flow cooled superconducting multi-strand cables. The model has been implemented in PSpice. While this work is focused on cable-in-conduit conductors (CICCs), the technique can readily be adapted to other types of superconducting multi-strand conductors or to multi-filamentary composites. In this article we report on the reproduction of experimental results to verify the model, which showed very good agreement. Furthermore, the influence of the magnetic self-field of a conductor and the impact of possible micro-defects in the superconducting filaments of a strand on the voltage-current characteristic have been studied by applying the presented model.

Index Terms — Coupled analysis, network model, superconducting CICCs, self-field effect, micro-defects.

I. INTRODUCTION

Cable-in-conduit conductors (CICCs) are used in superconducting large-scale applications to build magnets that produce time-varying magnetic fields, e.g. for large magnetic energy storage systems (SMES) or nuclear fusion experiments, see e.g. [1]. While benefiting from low eddy current losses, coils wound from multi-strand cables often yield a maximum transport current that is remarkably lower than the expected nominal current of the cable, i.e. the sum of the critical currents of all up to over 1000 strands. In many cases, the cable also exhibits a lower $n$-value (see Eq. (1)) than the virgin strands. Extensive work is in progress to investigate this degradation. Several mechanisms, that may lead to non-uniform current distribution in superconducting multi-strand cables, have been identified. Yet, their quantitative influence on the performance of real-scale conductors remains to be determined. On the one hand, detailed measurements of the processes inside a CICC are arduous, and perturbations of the conductor’s symmetry and the hydraulic conditions in the coolant flow caused by the instrumentation hardly can be avoided [2], [3]. Analytic investigations, on the other hand, are usually limited to simple two-strand conductor models [4], [5], [6]. Hence, numerical simulations are a promising tool, permitting a detailed look at strand currents and temperature and allowing reliably reproducible parameter studies for real-scale conductor designs.

Due to the temperature dependence of the critical current of superconductors, a comprehensive model has to comprise thermal analysis. Consequently, the electromagnetic model using lumped networks presented in [7] has been extended by a thermal and a hydraulic part [8].

However, modeling the current distribution inside a CICC at the present stage still cannot claim to give precision results for strand currents or temperatures. Too many parameters have to be roughly approximated, and many effects are still modeled in a rather simplified manner. In particular, modeling quench propagation is out of the scope of this work. The model is rather developed to provide insight on current distribution phenomena, helping to understand and evaluate the processes in actual-size CICCs.

The main features of the model are summarized in the following section, details can be found in [8]. After that, the validity of the model is verified by the reproduction of experimental results. In Section IV, we report on the application of the model to investigate the possible influence of micro-defects in the superconducting filaments on the voltage-current characteristic of a strand.

II. THE MODEL

Each of the $N$ strands of a CICC is modelled as a one-dimensional structure, i.e. the state variables like temperature or current density are assumed to be constant over the strand’s cross-section. The same principle is applied to the helium flow: To each strand, we associate a fraction of the total helium cross-section, that surrounds the strand and is modelled as a one-dimensional laminar flow. The combination of several 1D strand models and the associated He-cross-sections, that are galvanically, magnetically and thermally coupled, results in a quasi-3D model of a complete CICC. In many cases, treating several strands as macro-strands is reasonable, and can considerably reduce the model size.
The complete model was implemented as a representation as lumped network elements, which basically corresponds to a linear finite difference scheme. We used a commercially available software package featuring PSPICE, providing a state-of-the-art non-linear equation solver and time-stepping algorithm.

A. Electromagnetic Model

A superconducting multi-strand conductor can be regarded as a multi-conductor transmission line (MTL) with a considerable transverse conductivity and negligible capacitances between the strands. The governing equations and electric equivalent circuits are well-known [9]. However, the longitudinal resistive voltage, that diminishes as long as the strands are in the superconducting state, follows the empirical non-linear relation (power law, see e.g. [10])

\[ E = E_c \left( \frac{i}{I_c(B,T,\varepsilon)} \right)^n \]  

(1)

as the strand current \( i \) approaches the critical current \( I_c \) of the strand. For low temperature superconductors, usually a field criterion \( E_c = 10 \ldots 100 \) µV/m is used. The exponent \( n \) lies in the range of 10…35 for Nb$_3$Sn superconductors, in case of NbTi in the range of 20…50. For \( i \gg I_c \), the resistive voltage drop is dominated by the constant resistance of the copper stabilizer.

The critical current \( I_c(B,T,\varepsilon) \) is calculated as a function of the instantaneous values of the temperature \( T \) and the magnetic flux density \( B \), using the scaling laws proposed in [11] and [12] for Nb$_3$Sn and NbTi, respectively. The magnetic flux density \( B \) can be defined as a function of time, space, and the cable current, permitting to consider the self-field of a conductor and arbitrary background fields. For the dependence of \( I_c \) on the strain \( \varepsilon \) in case of Nb$_3$Sn, a constant value is used in the calculations.

B Thermal Model of a Strand

The thermal model of a single strand \( k \) to compute the local strand temperature \( T_k \) considers heat capacitance and longitudinal thermal conducance, and is based on the equation of energy conservation [13],

\[ c \cdot A_k \frac{dT_k}{dt} - \lambda \cdot A_k \frac{\partial^2 T_k}{\partial x^2} + h_k(t) \cdot p_{w,k} \cdot \left( T_k - T_{He,k} \right) = \dot{\Phi}_{joule,k} + \dot{\Phi}_{ext,k} \]

(2)

where \( c \) denotes the specific heat, \( \lambda \) the thermal conductivity, \( p_{w,k} \) the wetted perimeter, and \( A_k \) the cross-section of the strand. \( h_k(t) \) is a time-dependent coefficient that approximates the transient heat transfer from the strand to the surrounding helium of temperature \( T_{He,k} \) [8]. The heat sources are given by \( \dot{\Phi}_{joule,k} \), representing the Joule losses computed in the electromagnetic model, and \( \dot{\Phi}_{ext,k} \), which can be specified to account for external heat input, e.g. by friction losses or radiation.

C. Thermo-Hydraulic Model of the He-Flow

The model of the coolant flow associated to a strand \( k \), that is used compute the local helium temperature \( T_{He,k} \), is rather simplified. Friction and inertia are neglected, and the pressure is assumed to be constant. It is based on the one hand on the equation of energy conservation [13],

\[ \frac{\partial}{\partial t}(\rho_c C_p T_{He,k}) - \frac{\partial}{\partial x}(\rho_c C_p v_k T_{He,k}) + \sum_{j=1}^{N} k_{N,j}(T_{He,k} - T_{He,j}) = p_{w,k} h_k(t) \cdot (T_k - T_{He,k}) \frac{1}{A_k}, \]

(3)

where \( v_k \) is the flow velocity. The helium density \( \rho_c \) and specific heat \( C_p \), that determine the available enthalpy for cooling the strands, are modeled temperature dependent according to tabularized data [14]. The last term accounts for the transverse convective heat transfer to the local helium cross-sections of the adjacent strands \( j \), determined by an effective transverse heat conductance \( k_{N,j} \).

On the other hand, the model satisfies the continuity equation

\[ \frac{\partial}{\partial t} \frac{\partial}{\partial x} + \frac{\partial}{\partial x}(\rho_c \cdot v_k) = 0, \]

(4)

based on which the local flow velocity \( v_k \) is computed. Accordingly, the mass flow can vary locally due to thermal expansion of the helium.

D. Coupling of the Models

The three parts of the model are coupled both explicitly by relations between the state variables and source terms, and implicitly by temperature dependent material properties. A block diagram of the model of a segment of a single strand and its associated fraction of the coolant flow is shown in Fig. 1.

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**Fig. 1.** Computed quantities and coupling of the electromagnetic and thermal model of a strand \( k \) and the thermo-hydraulic model of the associated helium flow.
III. APPLYING THE MODEL TO CICCs

The validity of the electromagnetic part of the model has already been verified by comparison with experimental results, when the influence of the conductor’s symmetry on current distribution was investigated [7], [15]. The influence of the joint resistances and loop currents induced by magnetic fields has been studied in [8]. In this section, we will focus on the impact of the conductor’s self-field on the current distribution. The reproduction of the results of the CONDOPT experiment [17] is used to verify the thermo-hydraulic model as well.

A The Self-Field Effect

Important criterions of the performance of superconductors are the voltage-current and voltage-temperature characteristics, usually determined as $E(i)$- and $E(T)$-curves in a constant transverse background field $B_{\text{back}}$. But even for an ideally spatially homogeneous background field, there is a gradient of the magnetic flux density over the cable’s cross-section due to the self-field of the conductor, that superposes to $B_{\text{back}}$ [16]. The self-field varies between 0 in the cable axis and its maximum value

$$B_{\text{eff}}(R) = \mu \frac{J - R}{2}$$

on the outer radius $R$ of the conductor$^1$, where $\mu$ is the magnetic permeability. Hence, the magnetic flux density inside the cable becomes a function of the position in the plane perpendicular to the cable axis, $r = xe_x + ye_y$, and of the cable current $I_{\text{cable}}$. Due to the twist of the strands and subcables, $r$ in turn is a function of the longitudinal position $s$. The trajectory $r(s)$ of a (3x4)-subcable and the magnetic flux density along the longitudinal position $s$ of the ITER CS1 conductor [1] for example is shown in Fig. 2.

$$E_{\text{i}}(s) = I_{\text{i}}(B(r(s)), T(s), \varepsilon) .$$

Due to its field dependence, the critical current of a strand or subcable is a function of $s$ as well,

$$I_{\text{i}}(s) = I_{\text{i}}(B(r(s)), T(s), \varepsilon) .$$

Consequently, with rising cable current, the resistive voltage development starts locally in regions of higher field and lower $I_{\text{i}}$, see Fig. 3. Since the distribution of $E_{\text{long}}(s)$ differs for each strand or subcable, also a transverse voltage develops, driving an according transverse current between the strands. However, the simulations revealed, that even for unrealistic high values of the interstrand conductance $g$, the transverse current re-distribution is negligible and the subcable currents can be regarded as constant over $s$ (hence, strictly speaking, the self-field effect is no current distribution phenomenon).

Integrating $E_{\text{long}}(s, I_{\text{cable}})$ over the total length $l_{\text{tot}}$ of the conductor, and dividing by $l_{\text{tot}}$ yields the resulting $E(i)$-curve. Figure 4 shows curves as simulated with models where at each case a different cabling stage of the conductor has been lumped to macro-strands. It turned out, that even inside a (3x4x4)-subcable the self-field still has a considerable impact, and at least the (3x4)-subcables have to be modelled in detail for accurate results.

Figure 4 also contains the $E(i)$-curve, that would be expected from the extrapolation of the strand data, not considering the self-field. Applying a field criterion of $E_0 = 10 \mu \text{V/m}$, the self-field effect results in a critical cable current of 127.2 kA instead of 135 kA, which is degradation of 6%. In a logarithmic plot of Fig. 4 (not shown), the exponent $n$ in Eq. (1) can be determined. It turned out that $n$ is not affected by the self-field effect. This is not surprising, since due to the negligible transverse currents, all local voltage drops in the subcables evolve according to Eq. (1), i.e. they are proportional to $r$.

Consequently, the overall voltage development is also proportional to $r'$, with the original value for $n$. For a degradation of $n$, resistances with a current dependence of

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$^1$ Since the average current density $J$ is in the same range for all CICCs, Eq. (5) shows that the self-field effect is more pronounced in large conductors.
lower order than Eq. (1) must be present in the current path, see also Section IV.

The CONDOPT Experiment

The CONDOPT experiment in the SULTAN facility at CRPP on ITER sub-size conductors also revealed premature voltage development [17]. With the measured data, the results of the simulation of the experiment could be verified. A 6 m long hairpin-shaped sample of the SecA-conductor, a (3x3x4x4) CICC with ITER-type Cr-plated Nb$_3$Sn strands [18], was subjected to a 10 T background field over a length of 500 mm. We arbitrarily chose several measurements on the virgin conductor to compare with our simulation.

In the model, the (3x3) substages have been lumped to macro-strands, and the self-field was considered. The parameters for the Summers relations and further electrical and geometrical data were taken from previous measurements on straight samples of the SecA conductor [18]. While the virgin strands exhibited an $n$ exponent in the range of 30…35, in the model the value $n = 15$ determined for the cabled conductor sample was used, assuming that the $n$-value degradation rather stems from material effects during cabling than from electromagnetic phenomena (see also Section IV). The self- and mutual inductances have been calculated evaluating Neumann’s formula for low-frequency inductances over the trajectories of the twisted subcables [15].

Figure 5 shows the results for an $E(i)$-measurement with constant helium temperature and mass flow at the helium inlet of 5.41 K and 2.8 g/s, respectively. The current was ramped up with approximately 70 A/s.

In spite of the numerous simplifications in the model, the simulated results agree astonishingly well with the measurement. This holds especially facing the fact that an increase of the constant background field in the simulation of only 0.5%, which is in the range of the accuracy of the field measurement, results in exact congruence of the two curves. It must be stated that all parameters in the model have been specified according to the specifications of the conductor, i.e. no parameter was adjusted in order to fit the simulated results.

The simulated curve without self-field is also shown in Fig. 5. The decrease of the cable’s critical current due to the self-field effect amounts to 2.9% ($E_0 = 10 \mu$V/m).

Figure 6 shows the temperature rise of the coolant caused by the Joule power loss due to current sharing in the high-field region. Considerable heating starts around 15 kA, where the longitudinal electric field amounts already to more than 150 $\mu$V/m, i.e. beyond any common field criterion for the critical current. The simulated temperature lags behind the measured values. Since the calculation of the power of the heat sources in the model due to resistive losses is straightforward, the reason is probably rather an overestimation of the cooling of the conductor.

However, in view of the significant simplifications in the thermo-hydraulic model of the coolant flow, the results are still satisfying. In particular, the accuracy is sufficient to produce good results in the electromagnetic part of the model, where the primary focus of this work is on.
IV. APPLYING THE MODEL TO A MULTI-FILAMENTARY STRAND

With a few modifications, the model can also be applied to investigations of the current distribution among the superconducting filaments inside a single strand. In order to evaluate, how far micro-defects of the filaments inside the strands could be the cause of the degradation of the performance of a conductor, we investigated a scenario for an ITER-type strand, where 20% of the approximately 8000 Nb₃Sn filaments are affected by microscopic defects. These defects have been assumed to be sub-micrometer gaps in the filaments, bridged by the linear resistance of the copper stabilizer. In the model of a 100 mm strand section, we lumped all intact filaments to a single macro-filament. The defective filaments have been lumped to two branches, which had two gaps each at different locations and of different lengths, see Fig. 7. The thermo-hydraulic analysis was omitted in this simulation.

The inter-filament conductance $g$ was approximated by the copper resistance between filaments, reduced by a factor of 100 to account for the CuNi in the mixed matrix that hinders the transverse current transfer. The width of the gaps varied between 0.1 and 10 times an average gap length $l_{gap}$. Figure 8 shows the $E(i)$-characteristic of the strand for values of $l_{gap}$ in the range of 1 nm...10 µm. The curves for $l_{gap} = 1$ nm and $l_{gap} = 10$ nm are practically identical to the curve of the ideal strand.

The curves can be divided into three sections. For currents below 80% of the critical current of an ideal strand, the current flows completely in the intact filaments. The conductor behaves as if the defective filaments were not present.

Above 0.8*I$_{c}$, current sharing sets in in the intact filaments and drives the current more and more into the defective filaments, producing a voltage drop over the linear gap resistances. The combination of this linear voltage drop and the non-linear voltage drop according to Eq. (1) results in a reduced $n$-value of the $E(i)$-curves in this intermediate current range, which can be seen in the double-logarithmic plot in Fig. 9. Evaluating e.g. the curve for $l_{gap} = 1$ µm between $E = 10$ µV/m and $E = 100$ µV/m, we found an effective $n$-value of 18 instead of 35 for the ideal strand.

As the current rises further, the non-linear voltage drop becomes predominant both in the intact and the defective filaments, and the curves converge versus the characteristic of the ideal strand. It must be stated that depending on the average gap length, the critical current of the strand is not necessarily degraded significantly. For $l_{gap} = 0.1$ µm, for example, the transition in the intermediate current range is primarily below $i = I_{c}$, and the critical current is hardly decreased.

It was found that the transition in the intermediate current range becomes smoother, the more gaps with different gap lengths are present. Extrapolated to a real strand with thousands of filaments, we can assume an evenly distributed gap length, producing a perfectly smooth transition. Although there is no experimental evidence for the existence of such micro-defects so far, these investigations demonstrate that they constitute a potential cause for $n$-value degradation.

V. CONCLUSIONS

A detailed network model of forced-flow cooled superconducting multi-strand conductors has been presented. It comprises the coupled analysis of electromagnetic and thermo-hydraulic processes. The comparison of simulations and measurements yielded excellent agreement, although no model quantity was used as fitting parameter. While this may be fortunate...
coincidence and possibly not achieved in the same accuracy for other experiments, the model is clearly well suited for the intended purpose, i.e. the investigation and clarification of current distribution phenomena in CICCs.

Using the model, the influence of the self-field effect on the voltage-current characteristics has been investigated. It was found, that the critical current can be reduced by up to 6% for a large CICC like the ITER CS1 conductor, while the $n$ exponent remained unaffected. As a possible cause for $n$-value degradation, micro-defects in the superconducting filaments of a strand have been proposed and studied.

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REFERENCES


