Numerical Investigation of the Current Distribution in Cable-in-Conduit Conductors Using Lumped Network Models

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Abstract—Magnet coils wound from superconducting multi-strand cables exhibit a reduced current carrying capability during non-steady-state operation. This may be caused by a non-uniform current distribution in the cable. The extent of this non-uniformity depends on the design of the coil. Hence, measurements of the actual current distribution cannot be performed on short cable samples. In order to be able to estimate the performance of a cable for a given coil geometry already during the design phase of the magnet, lumped network models have been developed for a numerical investigation based on geometrical and material data of the coil and the cable. This work focuses on cable-in-conduit conductors (CICCs). Actual conductor designs have been modeled and the influence of the interstrand conductance and the geometrical accuracy of the cable was investigated.

Index Terms—CICC, current distribution, modeling, stability.

I. INTRODUCTION
Multistrand superconducting cables are used in large-scale applications to build magnets that produce time-varying magnetic fields, e.g., for nuclear fusion experiments or large magnetic energy storage systems. While benefiting from low eddy current losses, the maximum current of coils wound from multistrand cables often is remarkably lower than the expected nominal current of the cable, i.e., the sum of the critical currents of all strands. Recent experimental and theoretical work identify non-uniform distribution of the transport current among the strands as a possible cause [1], [2]. This may lead to the critical current of one strand to be exceeded, and to thermal power dissipation due to interstrand currents. Which of the two processes is predominant depends on the conductance between adjacent strands (interstrand conductance) and on thermal parameters. Since the effect does not occur for short cable samples, measurements are very costly, and, for large CICCs with up to over one thousand strands, practically impossible. We will show in this paper that the current distribution is extremely sensitive to perturbations of the cable’s geometry, which are hardly to be avoided when furnishing a coil with probes for measuring strand currents, temperatures or magnetic field. Meaningful experimental parameter studies are very difficult to realize due to the difficulties in exactly reproducing the same geometry for several test coils. Hence, it is very desirable to have simulation models for investigating the current distribution in coils wound from CICCs.

II. REASONS FOR CURRENT IMBALANCE
There are basically two reasons of current imbalance. One is an inhomogeneous distribution of the joint resistances of the strands at cable joints or current leads, that constitutes a resistive current divider. The resulting current distribution can be very easily determined once the joint resistances have been measured. The other reason is an inhomogeneous distribution of the time-derivative of the magnetic flux, dΦ/dt, enclosed by the various strands. This can be caused by either an inhomogeneous distribution of self- and mutual inductances, i.e., by geometrical distortion, or by large gradients of the magnetic flux density. The latter emerge e.g., in accelerator magnets and are not in the focus of this work.

The sensitivity of mutual inductances to geometric distortion can be demonstrated with two circular coaxial turns of the radii \( R_1 \) and \( R_2 \) and the distance \( h \). Their mutual inductance \( M_{12} \) is given by [3]

\[
M_{12} = \frac{2\mu_0 \sqrt{R_1 R_2}}{\sqrt{k_1}} \left( E(k_1) - K(k_1) \right)
\]

with \( k_1 = \frac{1 - \sqrt{1 - k^2}}{1 + \sqrt{1 - k^2}} \) and \( k = \frac{4R_1 R_2}{(R_1 + R_2)^2 + h^2} \).

\( K(k_1) \) and \( E(k_1) \) are the complete elliptic integrals of the first and second kind, respectively. The relative sensitivity \( \frac{dM_{12}}{d\lambda}/M_{12} \) of the mutual inductance in case of two turns of the same radius \( R \) to deviations of the scaled distance \( \lambda = h/R \) is shown in Fig. 1. Assuming a coil radius of 0.5 m and a strand diameter (that equals the distance between adjacent strands) of 0.5 mm, i.e., \( \lambda = 0.001 \), a deviation of \( h \) of only 10 µm results in a deviation of \( M_{12} \) of 0.29%. Numerical calculations with the magnetic field analysis software EFPI revealed a similar sensitivity for twisted strands.

The current distribution in case of inhomogeneous inductive coupling can simply be regarded as the result of an inductive current divider. However, the concept of loop currents (supercurrents [2]) that superpose to the transport current of the cable is very helpful to understand the process of current redistribution and the associated time constants, and phenomena such as V-ramping.
Fig. 1. Relative sensitivity of mutual inductance $M$ to the distance between two circular windings with same radius $R$. The distance is scaled to $R$, the sensitivity to $M$.

III. MODELING COILS WOUND FROM CICC

A CICC can be regarded as a multiconductor transmission line (MTL) [4]. While there is no reference conductor for the transport current, the total of the superposed loop currents is zero at any cross section of the cable. Thus, with the appropriate restrictions, the theory of MTLs can be applied to describe the supercurrents in CICCs. Fig. 2 shows the per-unit-length equivalent circuit of a CICC. At both ends of the cable the strands are soldered to a cable joint or a current lead, resulting in a very low joint resistance $R_{\text{joint}}$.

Considering the time-constants of involved currents and voltages in the range of seconds, displacement currents can be neglected. The series resistance $R'$ represents the non-zero resistivity of the strand in case the strand current comes into the range of the critical current $I_c$. For the perfectly superconducting state of the strands, i.e. for $i_s < I_c, R' = 0$, and the system of differential equations of the strand currents $\mathbf{i}$ in a uniform section of the CICC can be written as

$$\frac{\partial^2 \mathbf{i}}{\partial x^2} = G' \mathbf{M'} \frac{\partial \mathbf{i}}{\partial t}$$  \hspace{1cm} (2)

where $G'$ and $M'$ are the conductance and the inductance matrix (per unit length), respectively. Equation (2) describes only uniform conductors. A non-uniform conductor in a lumped circuit representation has to be approximated by a cascade of uniform sections. Regarding two strands in the frequency domain, the solution of (2) is

$$I(\omega, x) = I(\omega, 0) \cdot e^{-\gamma x} \hspace{0.5cm} \text{with} \hspace{0.5cm} \gamma = \alpha + j\beta = \sqrt{j\omega L' G'}$$  \hspace{1cm} (3)

where $\gamma$ is the propagation constant and $L' = L_1 + L_2 - M_1$ is the leakage inductance of the two strands. It is important to note that the time constants associated with current redistribution processes are not primarily determined by the total inductance of the coil, but in fact by the leakage inductance among the strands, which is in the range of $5\ldots40\%$ of the strands' self inductance. Nevertheless, for large coils wound from conductors with low interstrand conductance, the time constants can amount to several hours [2]. If the length of a conductor section is sufficiently shorter than the propagation length, $\gamma^{-1}$, for a given frequency, it can be modeled as a circuit of lumped elements.\footnote{Since (2) and (3) do not describe any wave propagation phenomena, it might be more meaningful rather to speak of an electric diffusion length $\alpha^2 = \sqrt{2/\alpha'(G' L')}$ (according e.g. to the thermal diffusion length).}

The self- and mutual inductances of the strands depend on the geometry of the eventual coil and can only be measured in case of insulated strands. Otherwise, it has to be numerically calculated, which requires a geometrical discretization of the CICC.

The interstrand conductance has to be measured with short cable samples. It strongly varies with transversal force (e.g. Lorentz force, bending force) and thermal treatment of the conductor. Hence, the measured values are only a rough approximation and subject to parameter studies.

The non-linear resistance per unit length $R'$ is constituted by the constant resistance of the copper matrix in parallel to the current-dependent resistance of the superconductor, that is given by the power law

$$R'(i_{\text{strand}}) = E(I_c) \frac{i_{\text{strand}}^{n-1}}{I_c^n}$$  \hspace{1cm} (4)

where $E(I_c)$ is the electric field in the superconductor for \(i_{\text{strand}} = I_c\). For NbTi, the exponent $n$ is typically in the range of 10...50.

IV. CONDUCTORS WITH INSULATED STRANDS

A. General Remarks

For conductors with $G' = 0$, the loop currents exclusively close via the joint resistances at the ends of the cable, and $\alpha \rightarrow \infty$ trends to infinity. Hence, the CICC behaves like a lumped circuit composed of $N$ parallel branches of one inductance in series with an $R_{\text{joint}}$ on either side. All inductances are magnetically coupled to another, resulting in $(2N^2-N)$ mutual inductances. The circuit constitutes a resistive current divider for steady state and very low frequencies or ramp-rates, respectively, and an inductive current divider for higher frequencies and ramp-rates. However, the transition frequency, proportional to $R_{\text{joint}}/L_i$, with $L_i$ in the range of mH and $R_{\text{joint}}$ in the range of some nΩ, is usually below 1 mHz. Hence, for practical operating frequencies and ramp-rates, the current distribution in multistrand conductors with insulated strands is independent of the ramp-rate.

B. Worked Example — a 3x3 CICC

The current distribution in a test coil wound from a NbTi 3x3-CICC was experimentally investigated in [5]. The self and mutual inductances were measured. The dispersion of the mutual inductances turned out to be $0.017\%$ for adjacent strands of the same triplet and $0.021\%$ for strands of different
triplets. This may well be in the range of the measurement accuracy, but realistic, as Section II points out. For measuring the strand currents, shunts of 61...79 \( \mu \Omega \) were inserted in each strand, i.e. the joint resistances had been artificially increased roughly by a factor of 1000. The analysis results of an appropriate SPICE model in the frequency domain is depicted in Fig. 3. The transition from resistive to inductive behavior is clearly to be observed at about 0.2 Hz. However, without the shunts, this frequency would be shifted to well below 1 mHz. The inhomogeneous mutual inductances cause a maximum strand current exceeding the mean value of as much as 24%.

Fig. 4 shows the analysis of a ramping process in the time domain. During ramping, self and mutual inductances determine the current distribution. After ramping, the induced loop currents decay with a time constant \( \tau_c \) of approximately 1.7 s, and we get the dc current distribution (however, without the shunts, this time constant would be well above 1000 s). Further analyses revealed that the ramp-rate is of no influence as long as the duration of the ramping process is shorter than \( \tau_c \).

C. Quench of a Single Strand

For a modification of the model we assumed that the critical current of strand #3 is being exceeded. The initial length of the normal zone was presumed to be twice the thermal diffusion length of the strand, i.e. approximately 50 mm.

The critical strand current is a function of the temperature \([6]\),

\[
I_c(T) = I_c(0\text{K}) \cdot (1 - (T/T_c)^2)
\]  

(5)

where \( T_c \) is the critical temperature for a given magnetic field (that is assumed to be constant). The block diagram of the non-linear resistance \( R_{SC} \) of the normal zone is given in Fig. 5. It was realized in SPICE using controlled voltage sources. The heat transfer coefficient \( h_{He} \) and the helium temperature \( T_{He} \) are assumed to be constant. Strand-to-strand heat transfer is not considered. The resulting strand currents and the critical current in the quenching strand #3 during ramping are shown in Fig. 6.

As the strand current of strand #3 approaches its critical value \( I_c \), the resistance \( R' \) of the to-be normal zone rises according to \( (4) \), and the copper matrix takes over a part of the strand current (current sharing). The resulting power dissipation heats the zone, and the critical current is being decreased, leading to a further rise of \( R' \). At a given point, this positive feedback leads to a sudden breakdown of \( I_c \), the strand quenches. It can be observed that 65% of the quenching strand’s current is commutated to the adjacent strands #1 and #2 of the same triplet. The other six strands are hardly affected at all. The commutation induces a spike on the terminal voltage of the coil. These voltage spikes have already been observed in experimental investigations by other researchers and associated to local quenches \([1]\).

After recovery of the normal zone, the strand current in strand #3 rises exponentially with a time constant of about 150 ms to its stationary value.

V. Conductors with Non-Insulated Strands

A. General Remarks

For \( G' > 0 \), loop currents do not close only via the joint resistances but also via the interstrand conductance, \( I_{0} \) becomes finite and the CICC behaves like a distributed parameter circuit. In order to represent it properly with lumped elements, the cable has to be divided in sections that are sufficiently shorter than \( I_{0} \) in the frequency range of interest. These sections, usually represented by \( \pi \)-elements, are considered to be uniform, i.e. to be regions of constant per-unit-length parameters.

B. Worked Example — the WENDELSTEIN 7-X Conductor

A test coil with 94 turns in 10 layers wound from a 200 m sample of the WENDELSTEIN 7-X conductor was investigated in \([7]\). The W7-X conductor is twisted of bare NbTi/Cu

\[
\begin{align*}
R_{SC} &= \text{resistance of superconductor} \\
I_{SC} &= \text{current through superconductor} \\
u_{IC} &= \text{voltage drop over normal zone} \\
T_{SC} &= \text{temperature of superconductor} \\
h_{HC} &= \text{heat transfer to helium} \\
C_{p} &= \text{heat capacitance of normal zone}
\end{align*}
\]
strands in four stages (3×4×4×4). The critical strand current $I_c$ is 183 A at 6.3 T and 4.2 K. An according SPICE model has been established. In order to reduce the size of the model, only one (3×4)-subcable has been modeled in detail, i.e. by one branch per strand. The other 180 strands have been represented by one single branch, assuming a uniform current distribution among them. Each layer was represented by one π-element.

As a starting point for parametrical studies, the resistance per unit length between strands and subcables has been measured with a reacted and cycled (and hence thoroughly oxidized) 1 m cable sample at room temperature and scaled to 4.2 K, yielding a value of 4500 Ω/m. The elements of the 130×130 inductance matrix have been numerically calculated using the magnetic field analysis software EFFI. The therefore required geometrical models of the multiple twisted strands were established using the specially developed preprocessor SUPERHELIX. To obtain inhomogeneous inductive coupling among the strands, the mutual inductances have been normally distributed with an arbitrarily chosen standard deviation of 0.2% for strands of the same triplet, 0.08% for adjacent triplets, and 0.03% else. These values represent a displacement of strands with a mean value of zero and a standard deviation of 8 μm. No non-linear resistances as described in Section IV.C have been implemented to the model so far due to convergence problems. The joint resistances have been set to 50 nΩ per strand. The coil was ramped at a rate of 1 kA/s to the theoretical nominal cable current of $192 I_c = 35$ kA.

We observed a maximum strand current in the cable that was 1.6 times the mean strand current. Assuming that the referring strand causes a complete quench, this would result in a quench current of the cable of 21.5 kA. In [7], a quench current of 13.63 kA was found, i.e. our model still overestimates the stability of the conductor.

A parameter variation of the interstrand conductance $G'$ revealed that the current distribution becomes more uniform for very high and for very low values of $G'$. While the former fact is well known and can be derived from a two strand model, the latter accounts for the statistical nature of the mutual inductances. For small values of $G'$ the strands are practically insulated, and the complete winding of a strand could be represented by a single π-element, whose inductance is the sum of the inductances of the sections. Since the mean value of the distribution of the mutual inductances is zero, the resulting inductance has a very small relative deviation. Consequently, the resulting inductance matrix is more uniform, and hence also the current distribution. However, this does not apply for steady state operation (i.e. when the current distribution is driven by the joint resistance distribution), and it would be different for a cable whose inductance matrix is systematically inhomogeneous, e.g. if one strand is weaker coupled to the others over the whole length of the coil winding.

VI. Conclusion

The analysis of lumped network models of coils wound from actual size CICC's yielded the following results:

1. The current distribution is extremely sensitive to inhomogeneity of the inductance matrix.

2. The mutual inductances, in turn, are extremely sensitive to perturbations of the cable geometry.

3. The transition from inductive to resistive current distribution takes place at very low frequencies or after a very long time, respectively. The time constants are determined by the leakage inductance and the interstrand conductivity among the strands, and the joint resistances.

4. For cables made of insulated strands, the ramp-rate does not influence the current distribution when being in the range of practical relevance.

5. The current of a quenching strand is mainly commuted to the adjacent strands of the same triplet. After recovery, it slowly takes up current.

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References


